FUNDAMENTAL STRINGS AS NONCOMMUTATIVE SOLITONS *

FINN LARSEN

Enrico Fermi Institute, University of Chicago
5640 S. Ellis Ave., Chicago, IL-60637, USA. †

The interpretation of closed fundamental strings as solitons in open string field theory is reviewed. Noncommutativity is introduced to facilitate an explicit construction. The tension is computed exactly and the correct spectrum is recovered at long wave length.

1. Introduction

It has recently been realized that strings and branes can be interpreted as noncommutative solitons in string field theory. The purpose of this talk is to review this development and provide a pedagogical introduction to the subject. The discussion is an extended version of the actual talk at STRINGS 2000 which in turn was based on[1].

String field theory is designed to mimic many aspects of ordinary quantum field theories. It is therefore natural to develop intuition about string field theory by applying standard field theory techniques in this less familiar context. This motivates the construction of classical soliton solutions in string field theory. In this talk the specific goal is to discuss a class of solitons related to the closed fundamental string. It turns out useful to introduce noncommutativity in the field theory, as a tool that facilitates an explicit construction. The result of the computation will be a soliton with tension exactly equal to the tension of the fundamental string; and it has the same classical fluctuation spectrum as well. These facts provide strong circumstantial evidence that the soliton can be *identified* with the fundamental string. In the course of the talk some open questions raised by this interpretation will be discussed. For definiteness the bosonic string theory is considered but virtually identical results apply to the superstring case.

2. Tachyon Condensation and a First Look at Solitons

Open string theory by definition supports open string excitations. Although it is not usually stressed, the ends of the open strings may be situated anywhere in

---

spacetime. Recalling that D-branes are defined as defects where open strings can end, the open string vacuum is thus characterized by a space-filling D-brane. In this terminology the 26-dimensional perturbative vacuum of open bosonic string theory is interpreted as a $D25$-brane. The spectrum of the open strings follows from standard string theory computations; the result is that the lightest mode is a tachyon, *i.e.* it has negative mass-squared. This means the potential of the tachyon field has negative second derivative in the perturbative vacuum. As is well-known (*e.g.* from the Higgs phenomenon in the standard model) this kind of tachyon signals an instability, there is a true vacuum where the "tachyon" field has acquired an expectation value. The driving force behind the developments in the last few years was an important insight by Sen[2], asserting that the true vacuum after tachyon condensation is in fact the standard perturbative closed string vacuum, *i.e.* the vacuum without the $D25$-brane and thus without the open strings. A consequence of this physical picture is that the energy liberated by the condensation of the tachyon precisely cancels the tension of the $D25$-brane.

We would like to develop a quantitative description of tachyon condensation in string theory. For fields with nonvanishing mass, such as the tachyon field, a constant field does not satisfy the equations of motion. It is therefore clear that tachyon condensation inherently involves off-shell properties. This is the reason that standard perturbative string theory is insufficient to analyze the problem, one must apply string field theory. The level truncation approximation to the cubic string field theory has provided convincing evidence that the energy liberated by tachyon condensation indeed equals that of the $D25$-brane[3]. This supports Sen's identification of the nonperturbative vacuum.

The interest in this talk is the more detailed question of *excitations* of the nonperturbative closed string vacuum; in particular, those adequately described as classical solitons. For definiteness the focus will be on the fundamental string excitations. Similar considerations apply to other soliton excitations, principally the lower-dimensional D-branes. The D-branes are in fact understood more precisely in this set-up; they are the topic of J. Harvey's talk at this conference.

The description of the fundamental string in open string theory is qualitatively as follows[4,5,6]. Consider first the situation *before* tachyon condensation. Then the open strings are described as some gauge field theory on the world-volume of the $D25$-brane. In this framework fundamental strings appear as electric flux-tubes. *After* tachyon condensation all open string degress of freedom are removed from the spectrum, and in particular the gauge field no longer exists. It is therefore not so obvious how to describe the electric flux-tubes after tachyon condensation. This is precisely the problem of interest because, whatever the appropriate description of electric flux-tubes after tachyon condensation, these are the fundamental strings. The whole process of tachyon condensation is reminiscent of the confinement of quarks: the open string degrees of freedom cannot propagate in the closed string vacuum but instead manifest themselves as collective excitations, such as the fundamental strings.

3. From Noncommutativity to a Quantitative Description

We would like to turn these comments into a quantitative field theory description. A suitable starting point for the discussion is the Born-Infeld Lagrangian[5]

$$S_I = - \int d^{26}x \ V(t) \ \sqrt{-\det[g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}]} \ . \tag{1}$$

One immidiate problem is that the tachyon potential $V(t)$ is unknown (except for a few qualitative features discussed in the previous section). A more pressing concern is that (1) is justified only for *constant* fields. In the complete Lagrangean the tachyon has kinetic terms, and there are numerous higher derivative terms that generally couple the tachyon and the gauge field. All these terms are determined *in principle* by string field theory. In practice they are unfortunately difficult to compute accurately. For example, an exact determination of these terms require that infinitely many massive fields of the string field are taken into account[‡]. The unknown derivative terms are important for fields varying over distances of order string scale. They are a serious problem for our purposes because the fundamental string solution we seek by definition varies over the string scale.

This is the point where noncommutativity turns out useful. Recall that a B-field can be incorporated in string theory by replacing the standard "closed string" metric $g_{\mu\nu}$ and coupling constant g_s with the "open string" quantities[9,10]

$$G_{\mu\nu} \;=\; g_{\mu\nu} - (2\pi\alpha')^2(Bg^{-1}B)_{\mu\nu} \ , \tag{2}$$

$$G_s \;=\; g_s \left(\frac{\det G}{\det(g + 2\pi\alpha'B)} \right)^{\frac{1}{2}} \ . \tag{3}$$

More importantly, one must also replace the standard multiplication of fields with the noncommutative star-product

$$A \star B = \exp\left(\frac{i}{2}\theta^{\mu\nu}\partial_\mu\partial_{\nu'} \right) A(x^\mu)B(x'^\nu) \ , \tag{4}$$

where

$$\theta^{\mu\nu} = -(2\pi\alpha')^2 \left(\frac{1}{g + 2\pi\alpha'B}B\frac{1}{g - 2\pi\alpha'B} \right)^{\mu\nu} \ . \tag{5}$$

The reason this is useful for the present problem is that the noncommutativity parameter θ provides a new scale in the theory. The linear extent of the fundamental string solitons we seek are of order string scale $l_s = \sqrt{\alpha'}$ when measured in the *closed string metric*; but for large θ/α' this corresponds to a distance $\sqrt{\theta}$ when measured in the *open string metric*. By way of comparison, the complications of open string field theory we want to control are of string scale with respect to the open string metric; so these are negligible on the scale of the soliton, as long as we take the limit $\theta/\alpha' \to \infty$. For related discussions see[11,12].

[‡]In the recent works[7] it was noted that the tachyon can be decoupled from the massive fields in the BSFT formalism of string field theory[8].

Before proceeding with the main line of development it is helpful to discuss the limit of large noncommutativity in more detail. Many workers (including Seiberg and Witten[10]) consider D-branes in background B-fields and take the low energy decoupling limit

$$\alpha' \sim \epsilon^{\frac{1}{2}} \to 0 \quad ; \quad g_{ij} \sim \epsilon \to 0 \ , \tag{6}$$

for i, j in the noncommutative directions. In this limit string theory reduces to noncommutative Yang-Mills theory (with some specific matter content). The decoupling limit (6) clearly implies large noncommutativity

$$\theta/\alpha' \sim \epsilon^{-\frac{1}{2}} \to \infty \ , \tag{7}$$

but it is *not* the limit we are considering. We take $\theta/\alpha' \to \infty$ *without* taking the low energy limit. This is important for our purposes because we want to keep string excitations. There are two dimensionless parameters in the problem $\alpha' E^2$ and θ/α'. The decoupling limit (6) takes $\theta/\alpha' \to \infty$ with θ, E fixed, and thus $\alpha' E^2 \cdot \theta/\alpha'$ fixed. In contract, we simply take $\theta/\alpha' \to \infty$ with $\alpha' E^2$ kept fixed. Our limit is that of noncommutative string field theory (NCSFT). As far as we are aware this limit has not been considered prior to[1].

We now return to the quest for a description of the fundamental string as a soliton by applying the limit of large noncommutativity to the Born-Infeld type Lagrangian (1). The string solution is going to be along some spatial direction, say x^1, as well as time x^0; we take large non-commutativity in all other directions. This introduces the open string metric G^{ij}, the open string coupling constant, and the star product in (1), yielding

$$S_I = -\frac{g_s}{G_s} \int d^{26}x \ V(t) \ \sqrt{-\det[G_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}]} \ . \tag{8}$$

The overall factor arises from the replacement $g_s \to G_s$ in the D-brane potential $V(t) \propto 1/g_s$. The progress at this point is that it is now justified to ignore derivatives along all transverse directions. Thus, for solutions independent of x^0, x^1, we can simply use (8) as it stands.

It is convenient for our purposes to consider the conjugate Hamiltonian

$$H = -\int d^{25}x \ \left[\sqrt{V(t)^2 + E^2/(2\pi\alpha')^2} + \lambda\partial_1 E\right] - \lambda p \ , \tag{9}$$

where the electric field E is the canonically conjugate of the gauge field A_1, and λ is a Lagrange multiplier imposing a restriction to the flux sector with quantum number p. The equations of motions become

$$\frac{V(t)V'(t)}{\sqrt{V(t)^2 + E^2/(2\pi\alpha')^2}} \ = \ 0 \ , \tag{10}$$

$$\frac{E}{\sqrt{V(t)^2 + E^2/(2\pi\alpha')^2}} + \lambda(2\pi\alpha')^2 \ = \ 0 \ . \tag{11}$$

These equations are quite simple. In fact, they appear too simple to allow nontrivial localized solutions. For example, the equations are obviously solved for tachyons with $V'(t) = E = \lambda = 0$ but this is only possible at the localized extrema of the potential and these solutions are therefore constant in spacetime. They are quite different from the *localized* solitons we seek.

At this point noncommutativity comes to the rescue again[13]. It is instructive to consider the equation

$$\phi * \phi = \phi \ . \tag{12}$$

The constant function $\phi = 1$ is the only solution, if $*$ is treated as the ordinary multiplication; but the noncommutative *-product involves infinitely many derivatives so the equation is actually a differential equation which may have nontrivial solutions. Indeed, there are many solutions[13]; the simplest is the Gaussian

$$\phi_0 = 2^{12} e^{-r^2/\theta} \ , \tag{13}$$

where $r^2 = (x^2)^2 + \cdots + (x^{25})^2$. Solutions to the equation $\phi * \phi = \phi$ are useful because functionals act on such functions in a simple way. For any functional f that can be expanded as a power series we have

$$f(a\phi) = \sum_{k=0}^{\infty} c_k a^k \phi^k = f(0) + [f(a) - f(0)]\phi \ , \tag{14}$$

where a is an ordinary number. As a result of this property the equations of motion become algebraic for *ansätze* built on solutions to $\phi * \phi = \phi$. It is therefore straightforward to find nontrivial solutions to the equations of motion.

4. The String Solutions

Let us consider some simple string solutions obtained this way. The simplest possibility is to take

$$t = t_* \phi_0 \ , \tag{15}$$

and other fields vanishing. Here t_* is chosen as the field at the perturbative extremum of the potential so that[13]

$$V'(t_* \phi_0) = V'(t_*)\phi_0 = 0 \ . \tag{16}$$

At large distances $\phi_0 \to 0$ so $t \to 0$ in the solution (15). In our conventions this corresponds to the nonperturbative vacuum. The solution (15) is interpreted as a D-string and discussed in more detail in[1].

It is simple to verify that

$$t = t_* \phi_0 \ ; \quad E = p\phi_0 \ , \tag{17}$$

satisfies (10-11) as well. Again, the solution is essentially the Gaussian (13); it is therefore fully localized and asymptotes the closed string vacuum at infinity. The

tension of the string soliton is determined from (9) as

$$T = \frac{1}{2\pi\alpha'}\sqrt{\frac{1}{g_s^2} + p^2} \ .$$ (18)

This result suggests that the solitonic string can be identified with the $(p, 1)$ string, *i.e.* the bound state of p fundamental strings and a D-string. Repeating the computation starting with other solutions to $\phi * \phi = \phi$ we find more general string solutions which can be interpreted as (p, q) strings with $q > 1$. The main goal is to find a solution with precisely the tension of the fundamental string, without any D-branes present. This seems to require a separate consideration. A candidate string solution with the correct tension $T = \frac{1}{2\pi\alpha'}$ is

$$t = 0 \quad ; \quad E = p\phi_0 \ .$$ (19)

Note that in each case discussed above the tension agrees *exactly* with the one known from perturbative string theory, even though the theory may not be supersymmetric.

The identification of (19) with the fundamental string is not entirely unproblematic. One issue is that the fundamental string tension is independent of g_s. This is puzzling because the action (1) depends on the coupling only through an overall factor $S \propto V(t) \propto 1/g_s$. The key feature that makes this possible is that $V(t) = 0$ in (19); this invalidates a simple scaling argument for the energy. The situation is similar to that of a massless particle with $V(t)$ playing the role of mass: the Lagrangian degenerates but the Hamiltonian presents no subtleties. Even though the tension computation is thus technically sound there is a cause for concern: $V(t) \to 0$ suggests that the effective coupling of the problem diverges, making quantum corrections important. Our understanding is that the correct loop counting parameter actually stays well-behaved so that quantum corrections are under control; however, this point deserves closer scrutiny.

Let us consider another issue. The solution (19) has the correct tension and electric flux to be identified with the fundamental string. The problem is that many other solutions have the same properties. Roughly speaking the equations of motion do not constrain the transverse profile of the solution at all. Specifially, if ϕ_k denotes a complete basis of solutions to (12) then the configurations

$$E = \sum_{k=0}^{\infty} a_k \phi_k \quad ; \quad a_k \in \mathbb{R} \ ,$$ (20)

all satisfy the classical equations of motion and could potentially be interpreted as the fundamental string.

General quantum properties of the underlying gauge theory improve the situation somewhat by requiring the a_k integral. Even so, countably infinite candidate fundamental strings remain. Fortunately this is not the end of the story: there is no conserved quantity preventing these configurations from mixing quantum mechanically. In fact standard maximally supersymmetric D-brane dynamics would provide

a unique quantum ground state. Unfortunately the situation is more involved here and the quantum problem cannot be analyzed precisely, but it may again have a unique ground state. The quantum problem deserves a better understanding.

In this talk we consider only the classical problem. Then the profile of the fundamental string is undetermined, it can spread out without violating any conservation laws. This is probably the correct result for infinite flux-tubes. To show flux confinement we need to add electric sources in bulk and show their flux escapes as a tube, rather than spreading out like a Coulomb field. It is possible that this can be understood already classically, as in[14], but we have not done so.

5. The Operator Formalism and Finite Noncommutativity

The presentation of the solutions above can be streamlined and the results significantly strengthened by introducing the operator formalism. The idea is to exploit the analogy between noncommutative geometry and the more familiar noncommutativity in quantum mechanics. The precise map between the two problems associates an operator $\hat{A}$ to each function A on the noncommutative space, and maps the noncommutative product $A * B$ to the more familiar operator product $\hat{A}\hat{B}$ in Hilbert space. In this way problems in noncommutative geometry translate into standard exercises in quantum mechanics.

Consider as an example the key equation $\phi * \phi = \phi$. In the operator formalism it reads $\hat{\phi}^2 = \hat{\phi}$ and therefore its solutions are simply the projection operators in Hilbert space. It is now clear that the equation has numerous solutions, indeed infinitely many. The soliton solutions presented in section 4 are thus essentially projection operators.

The operator formalism makes a huge symmetry manifest. Indeed, physics is left invariant under unitary transformations of operators and states

$$|\psi\rangle \rightarrow U|\psi\rangle \; ; \; \langle\psi| \rightarrow \langle\psi|U^\dagger \; ; \; \mathcal{O} \rightarrow U\mathcal{O}U^\dagger \; , \tag{21}$$

where

$$UU^\dagger = U^\dagger U = I \; . \tag{22}$$

These symmetries form the group $U(\mathcal{H})$. This points to a potential embarassment because it shows that any soliton solution in the theory has infinitely many "images" under $U(\mathcal{H})$. In many situations we want a unique soliton, to be identifed with its counterpart in closed string theory. The crucial observation is that the $U(\mathcal{H})$ is in fact a *gauge symmetry*[15]. The images under $U(\mathcal{H})$ are therefore not interpreted as distinct, but as gauge equivalent representations of a single physical state. In superstring theory the gauge symmetry is further enhanced by an infinite discrete group which removes certain tensionless solitons with no reasonable physical interpretation[16].

A key step in the discussion of section 3 was taking the limit $\theta/\alpha' \rightarrow \infty$ in order to justify neglecting derivative terms. We are now in a position to present an alternative argument, valid at any θ [17]. The objectionable derivative terms are in fact all

gauge covariant derivatives under the $U(\mathcal{H})$ symmetry. We can therefore imagine adjusting the gauge fields in the solution precisely such that the gauge covariant derivatives vanish identically, removing the need for neglecting them. That this is always possible relies on solution generating transformations of the form (21) but with (22) replaced by

$$UU^\dagger = I \ , \ U^\dagger U = I - P \ , \tag{23}$$

where P is some projection operator. We can therefore repeat the construction of solitons for finite θ [17]. This result is not surprising: in the closed string vacuum different values of the B-field are in fact gauge equivalent and it was therefore expected that vacua with different values of B are related. The argument above shows how this works in the open string variables by representing the noncommutative solitons as "almost" gauge equivalent to vacuum, at any θ.

6. Fluctuations

The noncommutative Born-Infeld type action (8) also describes long-wave length fluctuations depending on the commutative directions x^0, x^1. Allowing for these, the Hamiltonian (9) is replaced by

$$H = \int d^{25}x \left[\sqrt{E^\alpha M_{\alpha\beta}E^\beta + V(t)^2 \det(I+F)} + A_0 \nabla_\alpha E^\alpha \right] \ , \tag{24}$$

where

$$M_{\alpha\beta} = \delta_{\alpha\beta} - F_{\alpha\gamma}F^\gamma{}_\beta \ . \tag{25}$$

Using $t = 0 \Rightarrow V(t) = 0$ (closed string vacuum), $A_0 = 0$ (gauge condition), $F_{ij} = 0, F_{1i} = A'_i$ (derivatives negligible in NC directions) we find

$$H = \int d^{25}x \sqrt{(E^1)^2(1 + (\vec{A}')^2) + \vec{E}^2 + (\vec{E} \cdot \vec{A}')^2} \ , \tag{26}$$

where prime denotes the spatial derivative along the string and the vector notation refers to the transverse coordinates. The *ansatz* for a fluctuating string is

$$\phi = \phi_0(x^i - f^i(x^0, x^1)) \ , \tag{27}$$

$$E^1 = p\phi_0 \ , \tag{28}$$

$$\vec{E} = \vec{e}\phi_0 \ , \tag{29}$$

$$\vec{A}' = \vec{a}'\phi_0 \ . \tag{30}$$

We would like to find the effective action in $D = 1 + 1$ dimensions controlling the functions f^i. The Hamiltonian reduction procedure accomplishes this, with the result

$$H = \int dx^1 \sqrt{1 + \vec{\pi}^2 + (\vec{f}')^2 + (\vec{\pi} \cdot \vec{f}')^2} \ . \tag{31}$$

It is easy to compute the corresponding Lagrangian. In static gauge $X^\mu = (x^0, x^1, f^i)$ one finds

$$L = -\int d^2x \sqrt{(\dot{\vec{X}})^2(\vec{X}')^2 - (\dot{\vec{X}} \cdot \vec{X}')^2} \ . \tag{32}$$

This shows that *the effective action of long wave length fluctuations is the Nambu-Goto action with the correct tension!* The spectrum of the soliton is therefore precisely the same as for a fundamental string. If we take the action (32) seriously and quantize it we find *very* light excitations propagating along the string, including the graviton and even the closed string tachyon. These would appear here as collective excitations in open string field theory, a fascinating result. There is a standard objection against this line of reasoning: it is only the lightest objects in a theory that can be quantized and solitons must therefore usually be treated classically. This objection fails here because, unlike in many superficially similar computations, the fundamental string soliton is indeed the lightest excitation of the closed string vacuum, and therefore subject to quantization. It is nevertheless unjustified to trust the present computation beyond the long-wave length approximation: we are using here a crude effective action that only takes into account constant fields. It is justified to neglect derivatives in the transverse directions, because they are noncommutative, but fluctuations in the spatial and temporal directions remain. The effective action therefore applies only for long wave lengths satisfying $\sqrt{\alpha'}F'_{\mu\nu} \ll F_{\mu\nu}$. It is reasonable to expect that, in a better description, the fluctuations of the closed strings can justifiably be interpreted in terms of open string variables. The discussion of θ-dependence in section 5 may be a step in this direction.

7. Summary

We conclude with a brief summary.

- The starting point was the process of tachyon condensation in open string field theory, from the standard perturbative vacuum to the nonperturbative vacuum, identified with the closed string vacuum. Our interest was in soliton excitations of the nonperturbative vacuum.

- Noncommutativity was introduced in the system because it generates a new scale. In the limit of large noncommutativity it is justified to ignore higher derivatives and use a simple effective field theory description of the open string field theory. An argument for ignoring the derivatives even at finite θ was presented in section 5.

- An explicit construction of solitons identified with (p, q) strings follow. Solitons with the exact tension and long wave length fluctuations of fundamental strings were similarly constructed. These are identified with the closed fundamental strings expected in the nonperturbative vacuum.

- Several problems remain in the identification of noncommutative solitons and fundamental strings: the quantum properties of the effective gauge dynamics need better understanding, confinement must be elucidated already in the classical description, and the spectrum of fluctuations must be computed beyond the long wave length approximation.

Acknowledgements

References

1. J. A. Harvey, P. Kraus, F. Larsen and E. J. Martinec, "D-branes and strings as non-commutative solitons", JHEP **0007** (2000) 042, [hep-th/0005031].
2. A. Sen, "Descent Relations Among Bosonic D-branes", Int.J. Mod. Phys. **A14** (1999) 4061, [hep-th/9902105].
3. V. A. Kostelecky and S. Samuel, "On a Nonperturbative Vacuum for the Open Bosonic String", Nucl.Phys. **B336** (1990) 263; A. Sen and B. Zwiebach, "Tachyon Condensation in String Field Theory", JHEP **0003** (2000) 002, [hep-th/9912249]; N. Moeller and W. Taylor, "Level truncation and the tachyon in open bosonic string field theory", Nucl. Phys. **B583** (2000) 105, [hep-th/0002237].
4. P. Yi, "Membranes from Five-Branes and Fundamental Strings from Dp-Branes", Nucl. Phys. **B550** (1999) 214, [hep-th/9901159]; O. Bergman, K. Hori and P. Yi, "Confinement on the brane", Nucl. Phys. **B580** (2000) 289, [hep-th/0002223].
5. A. Sen, "Universality of the tachyon potential", JHEP **9912** (1999) 027, [hep-th/9911116].
6. J. A. Harvey and P. Kraus, "D-branes as unstable lumps in bosonic open string field theory", JHEP **0004** (2000) 012, [hep-th/0002117].
7. A. A. Gerasimov and S. L. Shatashvili, "On exact tachyon potential in open string field theory", [hep-th/0009103]; D. Kutasov, M. Marino and G. Moore, "Some exact results on tachyon condensation in string field theory", [hep-th/0009148], "Remarks on Tachyon Condensation in Superstring Field Theory", [hep-th/0010108].
8. E. Witten, "On background independent open string field theory", Phys. Rev. **D46** (1992) 5467, [hep-th/9208027]; S. L. Shatashvili, "Comment on the background independent open string theory", Phys. Lett. **B311** (1993) 83 [hep-th/9303143].
9. A. Connes, M. R. Douglas and A. Schwarz, "Noncommutative geometry and matrix theory: Compactification on tori", JHEP **9802** (1998) 003, [hep-th/9711162]; V. Schomerus, "D-branes and deformation quantization", JHEP **9906** (1999) 030, [hep-th/9903205].
10. N. Seiberg and E. Witten, "String theory and noncommutative geometry", JHEP **9909**, 032 (1999), [hep-th/9908142].
11. K. Dasgupta, S. Mukhi and G. Rajesh, "Noncommutative tachyons", JHEP **0006**, 022 (2000) [hep-th/0005006].
12. E. Witten, "Noncommutative tachyons and string field theory", [hep-th/0006071].
13. R. Gopakumar, S. Minwalla and A. Strominger, "Noncommutative solitons", JHEP **0005** (2000) 020 , [hep-th/0003160].
14. J. Kogut and L. Susskind, "Vacuum Polarization And The Absence Of Free Quarks In Four-Dimensions", Phys. Rev. **D9**, 3501 (1974).
15. L. Cornalba, "D-brane physics and noncommutative Yang-Mills theory", [hep-th/9909081]; N. Ishibashi, "A relation between commutative and noncommutative descriptions of D-branes", [hep-th/9909176].
16. J. A. Harvey, P. Kraus and F. Larsen, "Tensionless branes and discrete gauge symmetry", [hep-th/0008064].
17. J. A. Harvey, P. Kraus and F. Larsen, "Exact noncommutative solitons", [hep-th/0010060].